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ANALYSIS AND SYNTHESIS OF TAPERED FIN-LINES

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ABSTRACT

Fin-line tapers are useful for design of matching networks, filters, circulators, rectangular waveguide to fin-line transition etc. In this paper such tapers have been analysed and a synthesis technique presented using the closed-form expression for the power-voltage definition of the characteristic impedance. Reflection coefficient results are presented for exponential, parabolic and cosine squared tapers. Typical design data, using WR-19 housing and 150 μm RT - Duroid substrate, are presented for unilateral fin-lines.

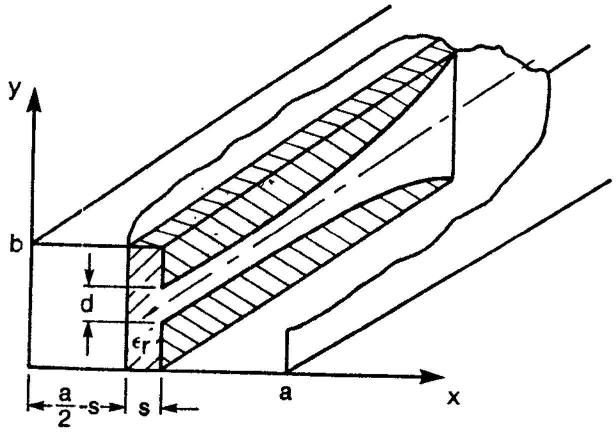

FIG. 1 - FIN-LINE TAPER

INTRODUCTION

Most practical E-plane circuit designs include one or several types of transitions, either between various types of fin-lines, or between fin-lines and different types of transmission line or microstrip.

Of particular importance is the transition between fin-line and the commensurate waveguide because of the need to interface fin-line circuits with conventional components and systems. One of the ways of realizing such transitions is with the use of tapered fin-line [1].

Analyses of tapered lines supporting a purely TEM mode deal with a constant propagation constant along the taper. Since the propagating constant varies along the length of a fin-line taper, the existing analyses [2-4], used for purely TEM lines, are inadequate in dealing with fin-line tapers. In the present work a Riccati equation is solved with the variation of propagation constant taken into consideration and thereby unilateral fin-lines, with exponential, parabolic and cosine squared tapers have been analysed. A technique has been presented for the synthesis of such tapers using closed-form equations developed earlier [5].

Input Impedance

Consider the equivalent circuit of the fin-line taper (Fig. 1) shown in Fig. 2. For the

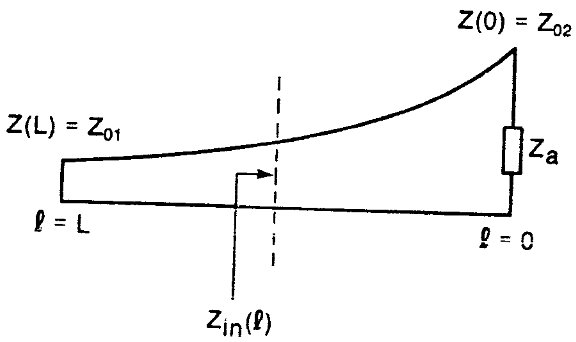

FIG. 2 - SCHEMATIC REPRESENTATION OF TAPERED TRANSMISSION LINE

input impedance Z_{in} at a distance ℓ from the load end and $Z_{in} + d\,Z_{in}$ at $\ell + d\ell$, assuming that the characteristic impedance $Z(\ell)$ and the propagation constant $\beta(\ell)$ remain constant over the incremental length $d\ell$, we have

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$$Z_{in} + d\,Z_{in} = Z(\ell)\,\frac{Z_{in} + j\,Z(\ell)\tan(d(\ell\beta(\ell)))}{Z(\ell) + j\,Z_{in}\tan(d(\ell\beta(\ell)))} \quad (1)$$

For small $d(\ell\beta(\ell))$, $\tan(d(\ell\beta(\ell))) = \beta(\ell)d\ell$, and hence

$$\frac{d\,Z_{in}}{d\ell} = j\,a(\ell) + j\,b(\ell)Z_{in}^2 \quad (2)$$

where $a(\ell) = Z(\ell)\beta(\ell)$ and $b(\ell) = a(\ell)/Z^2(\ell)$.

Equation (2) is a Riccati equation which can be solved numercially for a specified taper.

Reflection Coefficient

The reflection coefficient Γ and the input impedance Z_{in} at a given point ℓ on the line are related through the equation

$$Z_{in} = \frac{1+\Gamma}{1-\Gamma}\,Z(\ell) \quad (3)$$

Using equations (2) and (3) gives

$$\frac{d\Gamma}{d\ell} = \frac{1}{Z(\ell)}(j\,a(\ell)(1-\Gamma)^2 + j\,b(\ell)(1+\Gamma^2)Z^2(\ell) + (\Gamma^2 - 1)\frac{d\,Z(\ell)}{d\ell} \quad (4)$$

Again this can be solved on a digital computer using numerical techniques. $\beta(\ell)$, $Z(\ell)$ and $d\,Z(\ell)/d\ell$ are computed using the closed form equations developed.

To take the fin-line dispersion into account, an accurate determination of the guide wavelength λ_g may be made using the frequency dependent eqüivalent dielectric constant $k_e(f)$ and

$$\frac{\lambda}{\lambda_g} = [k_e(f) - (\lambda/\lambda_{ca})^2]^{1/2} \quad (5)$$

where

$$k_e(f) = k_c + \frac{k_1 - k_c}{\frac{b}{\lambda_1} - \frac{b}{\lambda_{cf}}}\left(\frac{b}{\lambda} - \frac{b}{\lambda_{cf}}\right) \quad (6)$$

and k_c is the equivalent dielectric cutoff of the fin-line, λ is the free space wavelength, k_1 is the value of $k_e(f)$ at the cutoff wavelength λ_1 of the air filled portion of the fin-line, λ_{cf} is the cutoff wavelength of the fin-line and λ_{ca} is the cutoff wavelength of the waveguide with the dielectric replaced by air. λ_1 is obtained from the LSE Model as

$$\lambda_1 = \lambda_{ca}\,\pi\,\sqrt{\bar{x}\,(3 - 2\bar{x})(\varepsilon_r - 1)/12} \quad (7)$$

$$\lambda_{cf} = \lambda_{ca}/\sqrt{k_c} \quad (8)$$

and $\bar{x}$ is obtained as a solution of the equation

$$\cot[\frac{\pi}{z}\bar{x}\sqrt{\frac{\varepsilon_r}{k_c}}] = \sqrt{\varepsilon_r}\,\tan[\frac{\pi}{z}(1-\bar{x})\sqrt{\frac{1}{k_c}}] \quad (9)$$

With this, the dispersion of the fundamental LSE mode is given by

$$\cot(\frac{\pi}{z}\bar{x}\frac{\lambda_{ca}}{\lambda}\sqrt{\varepsilon_r - \varepsilon_e(f)} = \sqrt{\frac{\varepsilon_r - \varepsilon_e(f)}{1 - \varepsilon_e(f)}}\,\tan[\frac{\pi}{z}(1-\bar{x})\frac{\lambda_{ca}}{\lambda}\sqrt{1 - \varepsilon_e(f)}] \quad (10)$$

where $\varepsilon_e(f) = \beta^2/k_0^2$ and β is the guide propagation constant.

RESULTS

Using equation (4) the reflection coefficient results are shown in Fig. 3, while design data are presented, for WR-19 housing with 150 µm RT - Duroid substrate in Fig. 4. The curves show that although the exponential taper gives the best performance it has the smallest fin-gap profile and hence, it is comparatively difficult to fabricate.

CONCLUSION

For the first time a systematic analysis of fin-line tapers has been done using closed-form equation for fin-line characteristics. Comparison among different types of taper shows the exponential taper to be the best.

ACKNOWLEDGEMENT

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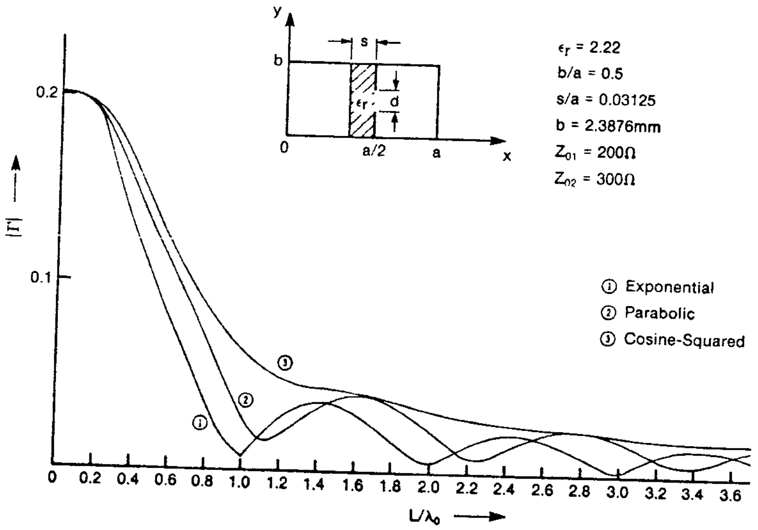

FIG. 3 - FREQUENCY RESPONSE OF TAPERED FIN-LINE

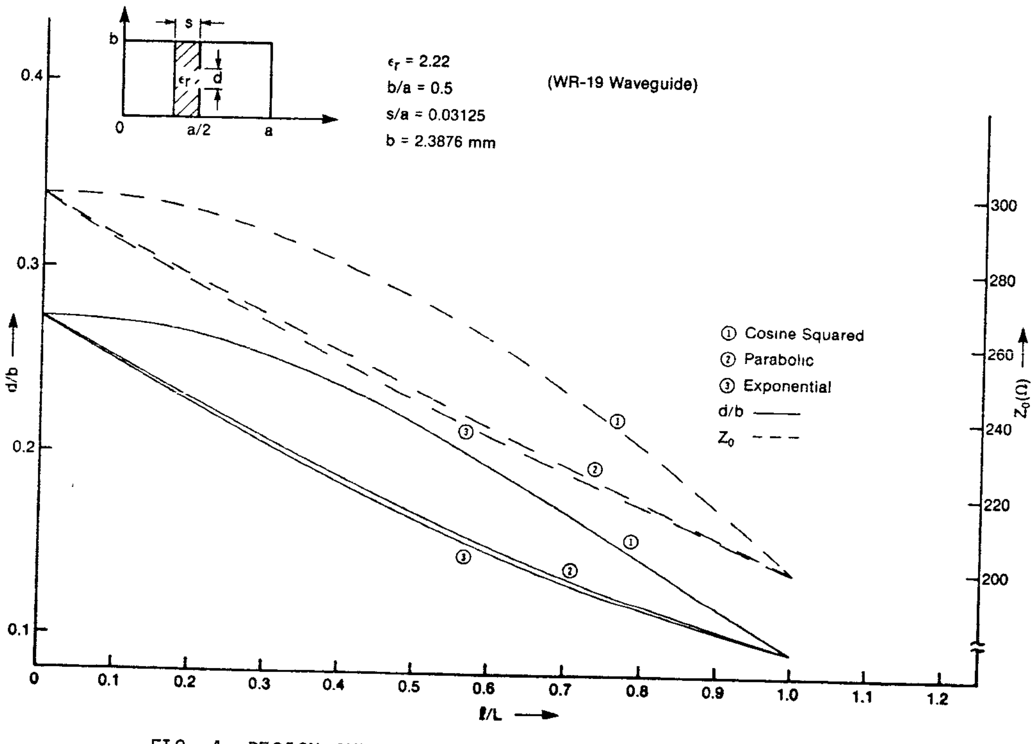

FIG. 4- DESIGN CURVES FOR FIN-LINE TAPERS.